A Short Treatise on Diminished and Augmented Scales and Chords

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1 Origin from the Even-Tempered Scale

As described in another document, the even-tempered scale is derived from finding the value which when taken to its twelfth power is exactly 2. This value is approximately 1.0595. Thus given the frequency of any even-tempered note may be multiplied by this value to get the next scale tone's frequency.

The rationale for this calculation is that an exact doubling or halving of the frequency of a note sounds similar and hence is called by the same name. For example, 'A' in concert tuning is 440hz. Hence, 880hz is also an 'A', as are 220hz and 110hz. Such a doubling is called an octave, based on the older notion that there were eight notes between any note and its double.

The evenness of this scale removes the problem of having intervals in a scale depend upon the starting note, as in the so-called "natural" scales.

Twelve, the number of notes in the half-tone octave scale, is divisible by itself (the trivial case), as well as 2, 3, 4, and 6. This large number of divisors creates the beauty of modern Western music. Across an octave from any starting note, then:

- Dividing the octave by 2 yields six notes which are a whole step apart.
- Dividing the octave by 3 yields four notes which are a minor third (b3) apart.
- Dividing the octave by 4 yields three notes which are a natural fourth apart.
- Dividing the octave by 6 yields 2 notes which are a flatted fifth (b5) apart.

Division by 2 yields 6 notes, which cannot generally be played together as a chord. These notes comprise the whole-tone scale.

Division by 3 yields four notes which when played together is called a diminished 7^{th} chord. In this document I will refer to this only as a diminished chord. The standard symbol for a diminished chord is a superscript 0, such as C^0.

Division by 4 yields three notes which when played together is called an augmented 5^{th} chord. In this document I will refer to this only as an augmented chord. The standard symbol for an augmented chord is a superscript +, such as D^+.

Division by 6 yields only two notes, so a chord is not possible using these notes.

The vital thing to recognize about diminished and augmented scales is that in any such given scale, all the intervals are identical. Practically, this means that each augmented or diminished chord is the augmented or diminished chord for each of its notes (four for diminished and three for augmented).

This also implies that all resolutions of which one augmented or diminished chord is capable apply to all the others whose note pattern it shares.

2 Diminished Chords

To build a diminished chord, start from any position and select ascending notes separated by a minor 3rd (three half-steps). There are only three such chords possible. Starting from 'C':

- C^0: C, D#/Eb, F#/Gb, A; hereinafter referred to as 1^0.
- $C\#^0$: C#/Db, E, G, A#/Bb; hereinafter referred to as 2^0.
- D^0: D, F, G#/Ab, B; hereinafter referred to as 3^0.

If we continue to Eb, the notes of Eb^0 will simply be an inversion of C^0, since all intervals are identical.

The 'textbook' method of creating a diminished chord is to play a dominant 7th (flatted 7th) chord and lower all but the root note by ½ step. The result is the same, just more confusing.

3 Augmented Chords

To build an augmented chord, start from any position and select ascending notes separated by a major 3rd (four half-steps or two whole notes). There are only four such chords possible. Starting from 'C':

- C^+: C, E, G#/Ab; hereinafter referred to as 1^+.
- $C\#^+$: C#, F, A; hereinafter referred to as 2^+.
- D^+: D, F#/Gb, A#/Bb; hereinafter referred to as 3^+.
- Eb^+: Eb, G, B; hereinafter referred to as 4^+.

If we continue to E, the notes of E^+ will simply be an inversion of C^+, since all intervals are identical.

The 'textbook' method of creating an augmented chord is to sharpen all 5th tones. The result is the same.

4 Diminished "Octatonic" Scales

The diminished or octatonic scales are created by interspersing the notes of any two distinct diminished chords in scale order. For example, merging 1^0 and 2^0, we get C, C#, D#, E, F#, G, A#, C. If we do this for all possible combinations of 1^0, 2^0 and 3^0 the pattern of intervals (in

half-steps) is always repeated; the only variation is whether we start with a whole step interval (pattern 2,1 repeated) or we start with a half-step (pattern 1,2 repeated).

Since there are only three possible combinations of 1^0, 2^0 and 3^0 there are exactly three distinct diminished scales, namely:

- $1^0 + 2^0$ :: 1^0 followed by ½ step, 2^0 followed by step
- $1^0 + 3^0$ :: 1^0 followed by step, 3^0 followed by ½ step
- $2^0 + 3^0$ :: 2^0 followed by ½ step, 3^0 followed by step

Other varieties of diminished scales are possible, but only by shifting the repeated 1, 2 at one or more points in the scale; hence, these are "decorated" or non-standard scales.

Using **C^0 as 1°, $C\#^0$ as 2^0 and D^0 as 3^0**, for example, we have:

Diminshed "Octotonic" Scales: interleaved diminished arpeggios					
Cdim+ 1/2 step, C#dim+step		**Cdim+step, Ddim+1/2 step**		**C#dim+1/2 step, Ddim+step**	
Notes	**1/2 Steps**	**Notes**	**1/2 Steps**	**Notes**	**1/2 Steps**
C	1	C	2	C#/Db	1
C#/Db	2	D	1	D	2
D#/Eb	1	D#/Eb	2	E	1
E	2	F	1	F	2
F#/Gb	1	F#/Gb	2	G	1
G	2	G#/Ab	1	G#/Ab	2
A	1	A	2	A#/Bb	1
A#/Bb	2	B	1	B	2
C		C		C#/Db	

These are the only such scales possible in standard tuning. Every other (non-identical) combination of diminished chords will yield one of these three scales.

5 Augmented "Tetrachord" Scales

The augmented or tetrachord scale is usually treated as being the same as the whole-tone scale; that is, the scale resulting from walking up the octave in whole-tone (2 half-step) intervals. It is considered an 'augmented' scale because every other scale tone makes up an augmented chord. (Remember that the scale positions of augmented chords are 4 half-steps apart.)

Since the interval pattern of a whole-tone scale is merely 2, 2, 2 repeated, there are only two whole-tone scales in the octave; namely, the one starting from 'C' and the one starting from 'C#'; all others are merely modes of these two.

Whole-tone scales can also be viewed as the result of interspersing the notes of two augmented chords. Specifically:

- C whole-tone scale is $1^{+} + 3^{+}$.
- C# whole-tone scale is $2^{+} + 4^{+}$.

While there are other possible combinations of the four augmented chords, they are not generally used, since the augmented chord "maps" so conveniently onto the whole-tone scale.

6 Altered or Diminished Whole-Tone Scale

The "altered" or "altered dominant" scale is a seven note scale derived from the Locrian mode. The easiest way to construct this scale is to take a standard major scale and lower all the non-tonic (non-root) scale positions by one half-step. For example, in 'C' we have:

C, C#, D#, E, F#, G#, A#, C

If you look at this scale, you'll see that the interior notes make up a B natural scale. So it can be viewed either as a flatted 'C' scale or as a 'B' scale with raised tonic.

This scale shares six of its seven notes with the diminished or octatonic scale.

7 Half-Diminished Scale

The half-diminished scale is a standard natural minor scale with a flatted fifth (b5). In 'C' this is:

C, D, Eb, F, Gb, Ab, Bb, C

If you separate the scale at its midpoint, you see that its first four notes are from the natural minor scale. In the latter half, the lowered 5^{th} (Gb) results in a series of whole-tone intervals, exactly the same as in a whole-tone or tetrachord scale.

8 Using Diminished Chords and Scales

Diminished chords are used in many types of music, particularly in the 'gypsy' styles of so-called 'hot' jazz and in much traditional music.

8.1 The Role of Diminished Chords

Diminished chords are usually viewed as dominant 7^{th} chords with a sharpened root. They then resolve to any of the tonics that relate to the dominant 7^{th} chord.

They can also be viewed as a root followed by a b3rd, b5th and bb7th (double-flatted 7th). They then resolve to the original chord with a normal 3rd, 5th, (and, optionally, b7th).

8.1.1 Diminished Chords as Dominant 7th Substitutions

For example, Eb^0 is viewed as an altered D7, since its notes are Eb, F#, A and C. These are the notes of a D7 with the root D raised to an Eb.

However, the same chord is likewise similar to an F7, Ab7 and B7; **just mentally lower each chord tone ½ step to find the related sevenths.**

Since the most common chordal resolution in music is that of the dominant 7th chord to the tonic (V -> I), we see that the Eb^0 chord, when considered as a dominant 7th, resolves readily to a G (from D7), Bb/A# (from F7), Db/C# (from Ab7) and E (from B7).

8.1.2 Diminished Chords as ½ Step Rising Resolutions

The 'standard' routine for creating a diminished chord is to play a dominant (flatted) 7th chord and lower all but the root. This, of course, implies that most of the notes readily resolve upwards ½ step to the original (unmodified) chord.

8.1.3 Diminished Chords as ½ Step Lowering Resolutions

8.2 Diminished Chord Substitutions and their Reasons

The most common substitution using a diminished chord is to replace a dominant 7th with the diminished chord for the sharpened root. Take the common progression:

G, G7, C

This becomes:

G, G#^{0}, C

Another common case is in minor keys, such as with

Em, B7, Em

becoming

Em, C^0, Em

The notes in G#^{0} are G#, B, D, F. These notes are a G7 with a raised root (G -> G#), so the chord resolves strongly to the sub-dominant in the key of G, which is C. But also note that it's similar to a Db^0 chord and contains the three non-root notes of a Db7 along with an actual D note. Since it's just a half-step from Db7 to D7 (the dominant 7th in G major), the G#^{0} also resolves strongly to D7!

Briefly then, you can inject a G#^{0} chord to pass to either the sub-dominant or dominant chord. The half-step resolution to D7 sounds a little 'jazzy', but that can be useful.

8.3 Key Modulation

Another common reason for using diminished chords is to modulate or transition between keys. In the above example we saw that G#^{0} resolves not only to C but to Eb, Gb and A. This resolution is not prejudicial; by this I mean that each such resolution is equally 'strong'. Although the listener may react to the change of key, the single chord transition from the diminished chord to the tonic of the new 'home' key (or its 5th chord) will not sound wrong.

Consider the following table.

Seventh Resolutions					
C^0		**C#^{0}**		**D^0**	
Pseudo 7ths	**Tonic Resolution**	**Pseudo 7ths**	**Tonic Resolution**	**Pseudo 7ths**	**Tonic Resolution**
B	E	C	F	C#/Db	F#/Gb
D	G	D#/Eb	G#/Ab	E	A
F	A#/Bb	F#/Gb	B	G	C
G#/Ab	C#/Db	A	D	A#/Bb	D#/Eb

Since there are only three distinct diminished chords, three columns suffice. There are two columns per chord. The "pseudo-7th" column lists the 7th chords that its diminished chord can 'masquerade' as. The "tonic resolution" column lists the V-I resolutions possible.

We take the G#^{0} example again, noting that it is the same as the third column or D^0. G#^{0} can masquerade as (be substituted for) Db7, E7, G7 or Bb7. Hence, using V-I resolution, it can be used to move to Gb, A, C or Eb, **either as a direct chord transition or an overt key change.**

Here are some simple derivation rules:

- The contents of each "pseudo 7th" column is the set of notes for each ***flatted*** tone in the diminished chord, which is the same as the notes in the diminished chord ***one half-step down***.
- The contents of the "tonic resolution" column is the set of chords for each ***sharpened*** tone in the diminished chord, which is the same as the notes in the diminished chord ***one half-step up***.

As a practical example, if you're playing a G major chord and briefly substitute a G#^{0} (acting as G7), you can now resolve to C, Eb, Gb or A through the magic of diminished V-I resolution. And don't forget D7, which resolves due to half-step similarity.

8.4 Improvising with Diminished Scales